of n_{O_2} equated to zero gives $x = C_2/C_1$. This value of x corresponds to a state of complete dissociation (maximum enthalpy). The zeros of the numerators of n_O and n_{O_2} are therefore the bounds of x. Calculations show the existence of not more than one root of Eq. (10) falling inside the determined bounds.

Inclusion of a neutral species, such as argon, would add one term in Eq. (7). Consequently, in the coefficients of the cubic, h would be replaced by $h - v_A n_A$. The algebra involved in carrying out the derivations becomes progressively more complicated as the number of reacting species is increased. The inclusion of ionization introduces higher-order algebraic nonlinearity. The authors have obtained explicit solutions in the case of coupled dissociation-ionization reactions (four species). Addition of molecular ionization, such as the $NO \rightleftharpoons NO^+ + e^-$ reaction, to the example discussed in this note leads to a sixth-degree polynomial in $y = (n_N/n_O)^{1/2}$, the roots of which have to be evaluated numerically. Ionization of atomic species in the presence of more than one dissociation reaction requires a simultaneous solution of two cubics. Thus the advantages of a closed-form solution are lost when the number of species is increased.

Although the existence of closed-form solutions is primarily of academic importance, the use of an exact solution, instead of an iteration scheme, results in considerable time savings in practical applications requiring a large number of computations.

Acceleration of Burning Rate of Composite Propellants by Sound Waves

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Nomenclature

ξ = particle displacement
p = acoustic pressure
ρ = gas density
c = gas-sound velocity
ω = angular sound frequency
δ = boundary-layer thickness
ν = gas-kinematic viscosity

Introduction

ONE serious limitation of solid-propellant rocket motors is that the thrust cannot be modulated upon command to meet in-flight contingencies. Thrust modulation by variation of nozzle-throat area, in a number of ways, has been under study for some time. An alternate approach to achieve thrust modulation is in-flight control of propellant burning rate. This paper is an account of an experimental study of using acoustics as a means for controlling the burning rate of composite solid propellants.

Theoretical Background

The concept of modulating the burning rate of composite solid propellants with sound waves is derived from the Summerfield[1] granular diffusion flame model.[1] It is hypothesized that pockets of gaseous fuel and oxidizer evolve from the hot propellant surface; the pockets then mix and react at some distance from the surface. This model suggests that controlling the rate of mixing of the gaseous pockets, under conditions where diffusion processes are rate-governing, would afford a means of controlling the burning rate. Plane travelling acoustic waves sweeping through the reaction zone would cause the gaseous pockets to follow the local oscillatory motion of the sound field so that more rapid mixing results. Accordingly, the burning rate should be accelerated, and modulation could be accomplished by variation of acoustic parameters.

There are three considerations that specify an appropriate acoustic field. One is that the sound field be of sufficient intensity so that the particle displacement can be large. In a plane travelling wave, the particle displacement is expressed by

$$\xi = p/\rho c\omega \qquad (1)$$

Since the oxidizer and fuel pockets differ in (ρc), the displacements of each for a given sound field will differ also. This relative motion between the pockets will give rise to enhanced mixing.

A second factor to be considered is the viscous forces between the acoustic motion and the channel walls. This gives rise to a reduction in particle displacement within an acoustic boundary layer, the thickness of which is given by

$$\delta = (2\nu/\omega)^{1/2} \qquad (2)$$

Efficient use of acoustic energy for mixing requires that the acoustic boundary layer be substantially less than the thickness of the reaction zone. This criterion imposes a lower limit on acoustic frequency according to Eq. (2). Finally, it is important that the gas pockets experience at least one complete oscillation while traversing the mixing zone. If one used estimated values for the pertinent combustion parameters it, was indicated in Refs. 1 and 2, that a sound field of 10 kc/sec at a sound pressure level of 174 db would be satisfactory.

Experimental Program

A schematic drawing of the experimental apparatus and instrumentation is shown in Fig. 1. All of the rocket-motor firings were made with internal-burning grains in chambers 5 in. in diameter by 20 in. in length, containing approximately 15 lb of an ammonium-perchlorate polyurethane-type composite propellant. Details of the instrumentation, firing procedure, and a sequential description of the test programs may be found in Refs. 2 and 3.

A Levavasseur whistle was used as the sound generator. Various gases under pressure could be fed through the whistle to provide the desired sound field. The output of the whistle was coupled to the rocket motor through an exponential horn to provide essentially plane wave propagation inside the motor cavity. Details of the whistle operation and performance are given in Ref. 4.

Experimental Results

Initially the sound field was produced by exciting the whistle with nitrogen gas at 9.6 and 14 kc/sec, each at 174 db. Thirty-nine firings were made with essentially no effect on the burning rate. It was decided that a gas having a

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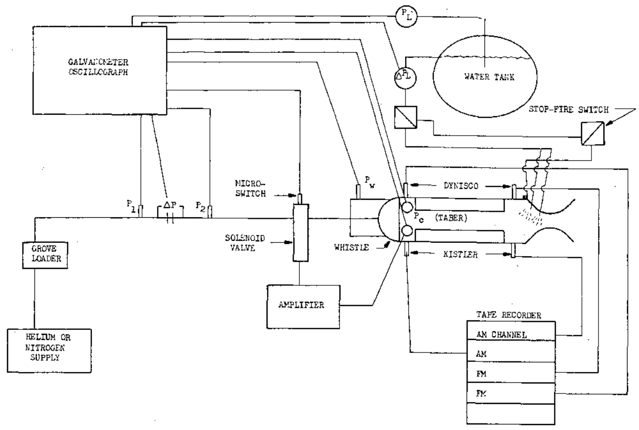

Fig. 1 Schematic of experimental arrangement and instrumentation.

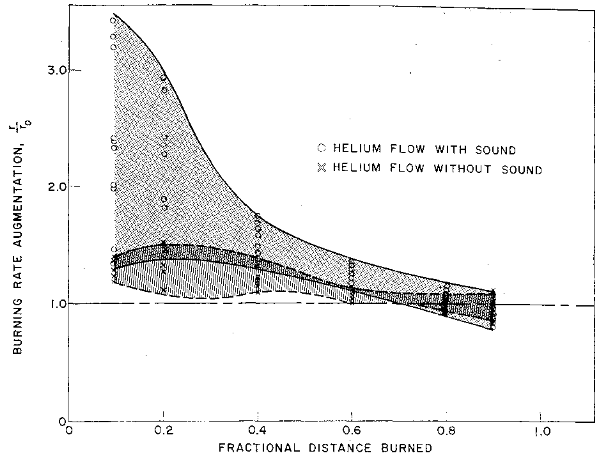

Fig. 3 Burning-rate augmentation by gas flow and sonic irradiation.

cold acoustic velocity closer to that of the hot combustion gases would be needed to prevent acoustic mismatching, and the whistle should be operated at a higher frequency to reduce the acoustic boundary-layer thickness. Helium gas was selected to satisfy both of these requirements.

Thirty-two firings were made subsequently. Positive results were obtained with helium exciting the whistle at frequencies of 30 and 35 kc/sec. Included were tare firings for each batch of propellant, as well as firings wherein an equal flow rate of helium was introduced into the chamber but with the sound-generating element of the whistle removed. Typical pressure-time curves are shown in Fig. 2; additional sets of curves and data from high-frequency response instrumentation are shown elsewhere.[2,3]

In every instance, the sonic irradiation was observed to produce a marked burning-rate increase early in the firing which tapered off to zero near burnout. The burning curves, expected to be progressive in accordance with the grain geometry, were thus given somewhat the characteristics of neutral-burning grains. A possible explanation on the basis of burning surface distortion was discounted by examination of extinguished grains. The most plausible explanation is as follows. An increase of the burning rate early in the firing caused the pressure to increase and, thereby, the reaction zone thickness to decrease; furthermore, the transpiration velocity of the gaseous pockets from the surface was also increased. These effects are the exact opposite of what is desired for the whistle to be effective. In other words, the burning-rate increase produced by the whistle was self-limiting.

The sound field produced by the whistle during motor firings exhibited amplitudes of 10 to 30 psi from peak-to-peak which are considerably higher than the values of 5 psi obtained from cold-flow calibration. Conceivably, there could have been an acoustic-combustion interaction. However, the whistle did not excite any acoustic instability mode of the motor.

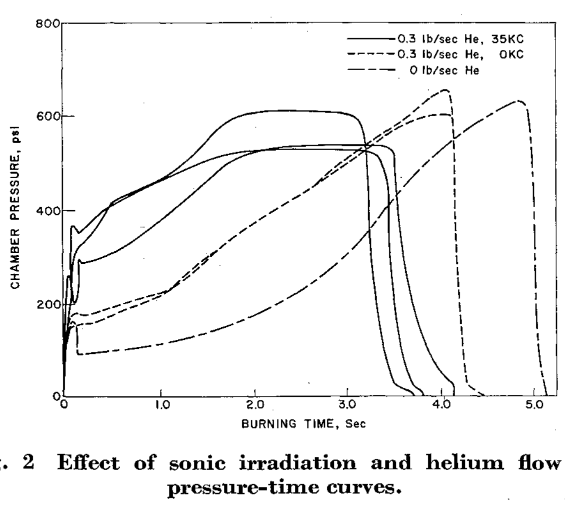

Fig. 2 Effect of sonic irradiation and helium flow on pressure-time curves.

The increase in burning rate observed for those firings with helium flowing through a silent whistle was not nearly as great as when sound was being produced. Figure 3 shows the burning-rate augmentations, calculated from the experimental data, as a function of the fractional distance burned. The effect of the helium flow without sound was greater than can be accounted for on the basis of the additional mass flow alone. Also the presence of helium must have caused an erosive effect through an increase of the molecular and eddy conductivities in the reaction zone. The effect of helium flow alone is seen also to diminish to zero near burnout. This is caused by the decreasing mass velocity of the helium with increasing port area; also, the mass contribution of helium to the total in the chamber becomes negligible near burnout.

There can be an erosive contribution to the burning-rate augmentation even when the whistle is producing sound. The sound field conceivably could be circulating the helium more vigorously in the turbulent boundary layer, encouraging still higher heat-transfer rates to the surface. However, the effect of sound was independent of helium flow rate within the range investigated (0.1 to 0.3 lb/sec). Additional experiments would be needed to clarify whether the effect of sound is chiefly through accelerated heat transfer or through accelerated mass diffusion as originally proposed.

A review of the pressure-time data (as also evident in Fig. 3) showed that the degree of reproducibility in the firings with sound was not as good as in those runs with helium flow but without sound. Possibly the effect of sound, strongly manifested during the initial portion of the firing, was influenced by the often variable ignition transient. Burning-rate augmentation by a factor as high as 3.4 was achieved with sound.

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